Problems 1 & 2

These two problems are the same as those in the last year's ssignment 3. Please refer to the supplement handout www.stanford.edu/class/cs206/hw5-sol2.pdf.

Problem 3

In general, computing the outcome of an auction requires exponential time. In this problem, we make use of the fact that only a bid of consecutive months is accepted. The following is the simulation of the operation of the $O(n^2)$ algorithm mentioned in the class:

Step 1: Bids for (Jan)

Bid 1: (Jan, $50) =	$50
Max(Jan) =	**$50**

Step 2: Bids for [Jan – Feb]

Max(Jan) + Bid 4: (Feb, $40) =	$90
Bid 2: (Jan – Feb, $80) =	$80
Max(Jan – Feb) =	**$90**

Step 3: Bids for (Jan – Mar)

Max(Jan) + Bid 5: (Feb – Mar, $110) =	$160
Max(Jan – Feb) + Bid 6: (Mar, $20) =	$110
Bid 3: (Jan – Mar, $150) =	$150
Max(Jan – Mar) =	**$160**

Hence, the winning bids are **Bid 1: (Jan, $50) and Bid 5: (Feb – Mar, $110)**.

Problem 4

a. The LP constraint for this problem in matrix form is as follows:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \text{x} \begin{bmatrix} a \\ b \end{bmatrix} \leq \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

where a and b (binary: 1 = yes, 0 = no) determine whether Alice or Bob (or both) gets their bids respectively. Notice that the order of the rows of the bid matrix does not matter in this case.

b. A matrix is TU if and only if the determinants of all the square sub-matrices are –1, 0, or 1. The bid matrix in the problem is TU because:
   1. the determinants of all the 1 x 1 sub-matrices are either 1 or 0.
   2. the determinant of the upper 2 x 2 sub-matrix is 1 while that of the lower 2 x 2 sub-matrix is –.