A Greedy Approach to Combinatorial Auctions

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Greedy allocation scheme

- Assume single-minded bidders (one bid per bidder)
- Order bids by descending average price per good
- Allocate greedily going down the list
- Each winning bid pays
  (number of goods in bid) *
  (average-price-per-good in highest displaced bid)
- Losing bidders pay nothing

Example

- Bids: Red bids 10 from {a}, Green bids 19 for {a,b}, Blue bids 8 for {b}
- GVA: Green gets {a,b}; pays 10 to Red, 8 to Blue
- Greedy allocation: Red gets {a}, Blue gets {b}
- GVA-style payment: Red pays 19-8=1, Green and Blue pay nothing. Note: Red's payoff negative.
- New-style payment: Red pays 9.5, Green and Blue nothing

Why study this greedy method?

- A simple test case
- 1/k lower-bound on the approximation ratio 1/k, where k is number of goods
- Can replace scoring function a/s by a/?s, in which case lower bound drops to 1/?k
- In practice usually better
- Proof techniques very general and apply to other Cas
- In particular, to an anytime version of the greedy method

Sufficient conditions for incentive compatibility with single-minded bidders

1. Exactness: You get what you asked for or you get nothing
2. A-monotonocity: If you win, you'd also win if you asked for less or offered more money or both

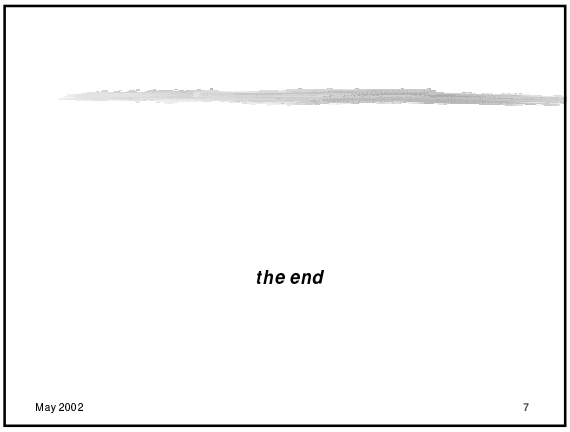
Lemma: Exactness and A-monotononicity imply the existence of V_c, above which you win and below which you lose

3. Criticality: If you win you pay V_c
4. P-monotonicity: If you win, you'd pay less if you asked for less
5. Participation: if you don't win you pay nothing

Theorem: Exactness, A-monotonocity, Criticality, P-monotonicity and Participation ensure incentive compatibility

Further truths

- The greedy scheme is incentive compatible for single-minded bidders
  - Proof: it satisfies the axioms
- An "anytime" CA
  - Perform chronological backtracking to find better solutions (in the limit, the optimal one)
  - At each level can stop the algorithm and calculate new-style payments
  - Axioms still hold, hence does incentive compatibility
- Beyond single-minded bidders
  - No payment scheme can make the greedy mechanism incentive compatible for complex bidders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